

# ONE-LOOP QCD INTERCONNECTION EFFECTS IN PAIR PRODUCTION OF TOP QUARKS

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## ABSTRACT

We calculate the one-loop non-factorizable QCD corrections to the production and decay of pairs of top quarks at various collider experiments. These non-factorizable corrections interconnect the different production and decay stages of the off-shell top-pair production processes. This in particular affects the invariant-mass distributions of the off-shell top quarks, resulting in a shift of the maximum of the distorted Breit-Wigner distributions. Although the non-factorizable corrections can be large, the actual shift in the mass as determined from the peak position of the corrected Breit-Wigner line-shape is below 100 MeV.

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# 1 Introduction

At present and future collider experiments, a detailed investigation of the production of top-quark pairs will substantially contribute to our knowledge of the top-quark properties and thereby of the Standard Model. An improved measurement of the top-quark mass  $m_t$ , for instance, can serve to obtain improved indirect sensitivity to the mass of the Standard Model Higgs boson. This is achieved by combining the high-precision measurements of the electroweak parameters at LEP/SLC with the direct measurements of the top-quark and  $W$ -boson masses.

Pairs of top quarks can be produced in hadron collisions at the Tevatron ( $p\bar{p}$ ) and LHC ( $pp$ ), as well as in  $e^+e^-$  and  $\gamma\gamma$  collisions at a future linear collider. Since the top quark has a large width as compared to the QCD hadronization scale,  $\Gamma_t \approx 1.4 \text{ GeV} \gg \Lambda_{\text{QCD}} \approx 200 - 300 \text{ MeV}$ , it predominantly decays before hadronization takes place. Therefore the perturbative approach can be used for describing top quarks. The main lowest-order (partonic) mechanisms for the pair production of top quarks are

$$e^+e^-, \gamma\gamma \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow 6 \text{ fermions}, \quad (1)$$

$$q\bar{q}, gg \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow 6 \text{ fermions}. \quad (2)$$

A lot of effort has been put into an adequate theoretical description of these reactions (see e.g. Ref. [1] for two review papers). Most of these studies treat the top quarks as stable particles, which is a reasonable approximation since  $\Gamma_t/m_t = \mathcal{O}(1\%)$ . For the reactions  $q\bar{q}, gg \rightarrow t\bar{t}$  these studies comprise QCD [2] and electroweak [3] one-loop corrections, as well as the resummation of soft-gluon effects [4]. Also for the reactions  $e^+e^-, \gamma\gamma \rightarrow t\bar{t}$  both the QCD [5] and electroweak [6] one-loop corrections are known. Moreover, the  $t\bar{t}$  threshold, with its sizeable QCD [7] and Yukawa interactions [8], has been analysed in detail.

One would, however, like to treat the top quark as an unstable particle, with a Breit–Wigner distribution describing its line shape. The most economic approach for treating processes that involve the production and subsequent decay of unstable particles is the so-called leading-pole approximation (LPA) [9]. This approximation is based on an expansion of the complete amplitude around the poles of the unstable particles, which can be viewed as a prescription for performing an effective expansion in powers of  $\Gamma_i/M_i$ . Here  $M_i$  and  $\Gamma_i$  stand for the masses and widths of the various unstable particles. The residues in the pole expansion are physically observable and therefore gauge-invariant. The actual approximation consists in retaining only the terms with the highest degree of resonance. In the case of top-quark pair production only the double-pole residues are hence considered and the LPA becomes a double-pole approximation (DPA). This approximation will be valid sufficiently far above the  $t\bar{t}$ -threshold. If in reactions (1) and (2) also the  $W$  bosons are treated as unstable particles, then also for these particles the leading pole residues should be taken. In this approach the complete set of corrections to reactions (1) and (2) naturally splits into two groups: factorizable and non-factorizable corrections. The factorizable corrections are directly linked to the density matrices for on-shell production and decay of the unstable particles. The non-factorizable corrections can be viewed as describing interactions that interconnect different (production/decay) stages of the off-shell process. A detailed discussion of this method with all its subtleties can be found in

Ref. [10], where the method has been applied to the complete set of  $\mathcal{O}(\alpha)$  radiative corrections to the process  $e^+e^- \rightarrow W^+W^- \rightarrow 4$  fermions. For  $t\bar{t}$  production partial results along this line exist [11], involving a subset of the factorizable corrections to the reaction  $e^+e^- \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow 6$  fermions. However, the non-factorizable corrections are needed for a complete  $\mathcal{O}(\alpha_s)$  calculation.

In recent years the necessary methods for calculating such non-factorizable corrections have been developed. In Ref. [12] a first complete calculation was performed for  $t\bar{t}$  production in  $e^+e^-$  collisions. In Ref. [13] the non-factorizable corrections were calculated for  $W$ -pair production, revealing differences with the results of Ref. [12]. The results of Ref. [13] were confirmed by an independent calculation [14] as well as by a re-analysis of the results of Ref. [12].

In this paper we apply our calculations presented in Ref. [13] to the non-factorizable  $\mathcal{O}(\alpha_s)$  corrections to  $t\bar{t}$  production at various colliders. We discuss the effect on the invariant-mass distribution of the off-shell top quark and the resulting shift in the maximum of the distorted Breit–Wigner distribution.

## 2 Definition of the non-factorizable corrections

In the LPA approach reactions like (1) and (2), which involve unstable particles during intermediate stages, can be viewed as consisting of separate subprocesses, i.e. the production and decay of the unstable particles. Having this picture in mind, the complete set of radiative corrections can be separated naturally into a sum of corrections to these subprocesses, called factorizable corrections, and those corrections that interconnect various subprocesses, called non-factorizable corrections. It should be noted, however, that it is often misleading to identify the non-factorizable contributions on the basis of diagrams. Such a definition is in general not gauge-invariant. Rather one should realize that only real/virtual semi-soft gluons<sup>1</sup> with  $E_g = \mathcal{O}(\Gamma_i)$  will contribute, the contributions of the hard gluons being suppressed by  $\Gamma_i/E_g$ . This is a consequence of the fact that the various subprocesses are typically separated by a big space-time interval of  $\mathcal{O}(1/\Gamma_i)$  due to the propagation of the unstable particles. The subprocesses can be interconnected only by the radiation of semi-soft gluons with energy of  $\mathcal{O}(\Gamma_i)$ , which induce interactions that are sufficiently long range. Hard gluons ( $E_g = \mathcal{O}(M_i) \gg \Gamma_i$ ) as well as massive particles induce short-range interactions and therefore contribute exclusively to the factorizable corrections, which are governed by the relatively short time interval  $\sim 1/M_i$  on which the decay and production subprocesses occur. A more detailed discussion of these issues can be found in Refs. [10, 15, 16].

In Fig. 1 we show schematically the partonic process  $q\bar{q} \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow 6$  fermions. The process consists of five subprocesses, which we will denote by  $t\bar{t}_{\text{prod}}$ ,  $t_{\text{dec}}$ ,  $\bar{t}_{\text{dec}}$ ,  $W_{\text{dec}}^+$ , and  $W_{\text{dec}}^-$ . In Fig. 1 these subprocesses are indicated by the open circles. The non-factorizable semi-soft gluon interactions interconnect any two different subprocesses, as is exemplified in

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<sup>1</sup>These gluons will still be perturbative in our case as their typical energy ( $E_g \sim \Gamma_{t,W} \gtrsim 1.4 \text{ GeV}$ ) largely exceeds the QCD hadronization scale ( $\Lambda_{\text{QCD}} \approx 200 - 300 \text{ MeV}$ ).

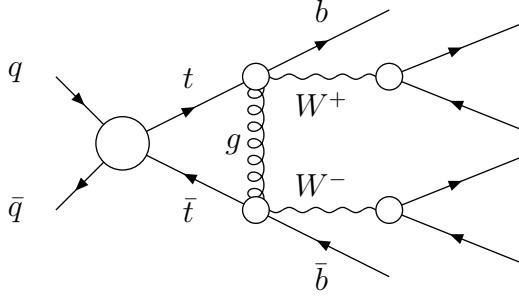


Figure 1: The generic structure of the complete  $t\bar{t}$ -production process  $q\bar{q} \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow 6$  fermions in the LPA. The open circles denote the various production and decay subprocesses. As an example also the non-factorizable semi-soft gluon interaction between the two top-quark decay subprocesses is shown.

Fig. 1 for the two top-quark decay subprocesses. The coupling of such a gluon to a certain subprocess can be written in terms of semi-soft currents. In contrast to soft-gluon currents, the effect of the gluon momentum on the unstable-particle propagators cannot be neglected in the semi-soft currents. The various non-factorizable corrections to the cross-section are just given by all possible interferences of the semi-soft currents. This will be made more explicit in the next section.

### 3 Colour dependence of the non-factorizable corrections

We start off by considering the simpler case of stable  $W$  bosons. At the end of this section we will indicate what happens if the  $W$  bosons decay hadronically. For stable  $W$  bosons one can identify three subprocesses:  $t\bar{t}_{\text{prod}}$ ,  $t_{\text{dec}}$ ,  $\bar{t}_{\text{dec}}$ . The non-factorizable corrections are given by the semi-soft gluon interferences between these different subprocesses. As only semi-soft gluons contribute, the virtual and real matrix elements factorize in terms of lowest-order matrix elements and semi-soft currents. In view of the possible presence of coloured particles in the initial state ( $q\bar{q}, gg$ ), this factorization depends on the colour structure. For the reactions (1), which involve only colourless initial-state particles, the  $t\bar{t}$  pair is produced in a singlet state. In contrast, the  $t\bar{t}$  pair is produced in an octet state in the lowest-order annihilation process  $q\bar{q} \rightarrow t\bar{t}$ , which involves the time-like exchange of a gluon. Both singlet and octet states are present in the lowest-order gluon-fusion reaction  $gg \rightarrow t\bar{t}$ , since in that case also space-like top-quark-exchange diagrams contribute. Because of these differences in the colour structure of the lowest-order reactions, also the non-factorizable corrections will come out differently, as we will see from the following discussion.

In order to keep the notation as general as possible, we write the lowest-order partonic reactions in the generic form

$$Q_1(q_1)Q_2(q_2) \rightarrow t(p_1)\bar{t}(p_2) \rightarrow b(k_1)W^+(k'_1)\bar{b}(k_2)W^-(k'_2), \quad (3)$$

where  $Q_1 Q_2 = \{e^+ e^-, \gamma\gamma, q\bar{q}, gg\}$ . The corresponding lowest-order matrix element will be denoted by  $(\mathcal{M}_0)_{ij}^{c_2 c_1}$ , where  $i, j$  indicate the  $t, \bar{t}$  colour indices in the fundamental representation. The colour indices  $c_1, c_2$  belonging to  $Q_1, Q_2$  depend on the specific initial state: they are absent for the colourless  $e^+ e^-$  and  $\gamma\gamma$  initial states, and they are in the fundamental/adjoint representation for the  $q\bar{q}/gg$  initial states. The momentum, Lorentz index, and colour index of the semi-soft gluon will be denoted by  $k, \mu$ , and  $a$ , respectively.

By using the relation

$$(T^a)_{ij}(T^a)_{kl} = \frac{1}{2} \left( \delta_{il} \delta_{kj} - \frac{1}{N} \delta_{ij} \delta_{kl} \right) \quad (4)$$

for the  $SU(N)$  generators  $T^a$  in the fundamental representation (with  $N = 3$  for QCD), the virtual and real non-factorizable corrections take the generic form:

$$d\sigma_{\text{nf}}^{\text{virt}} = \frac{1}{K_{\text{in}}} \frac{d\Gamma_0}{2s} (\mathcal{M}_0^*)_{i''j''}^{c_2''c_1''} (\mathcal{M}_0)_{i'j'}^{c_2'c_1'} \text{Re} \left\{ i \int \frac{d^4 k}{(2\pi)^4 [k^2 + i0]} (\Delta_{\text{nf}}^{\text{virt}})_{i''i';j'j''}^{c_2''c_2';c_1'c_1''} \right\}, \quad (5)$$

$$d\sigma_{\text{nf}}^{\text{real}} = - \frac{1}{K_{\text{in}}} \frac{d\Gamma_0}{2s} (\mathcal{M}_0^*)_{i''j''}^{c_2''c_1''} (\mathcal{M}_0)_{i'j'}^{c_2'c_1'} \text{Re} \left\{ \int \frac{d\vec{k}}{(2\pi)^3 2k_0} (\Delta_{\text{nf}}^{\text{real}})_{i''i';j'j''}^{c_2''c_2';c_1'c_1''} \right\}. \quad (6)$$

Here the pre-factor consists of the lowest-order phase-space factor in the LPA  $[d\Gamma_0]$ , the partonic flux factor  $[1/(2s)]$ , and the initial-state spin and colour average  $[1/K_{\text{in}}]$ . The non-factorizable kernels can be expressed in terms of semi-soft currents according to

$$\begin{aligned} (\Delta_{\text{nf}}^{\text{virt}})_{i''i';j'j''}^{c_2''c_2';c_1'c_1''} &= \frac{1}{2} \delta_{c_2''c_2'} \delta_{c_1'c_1''} \left\{ \delta_{i''j''} \delta_{j'i'} \left[ \mathcal{J}_t^\mu (\mathcal{J}_{t\bar{t}} - \tilde{\mathcal{J}}_{t\bar{t}} - \tilde{\mathcal{J}}_\oplus)_\mu + \mathcal{J}_{\bar{t}}^\mu (\mathcal{J}_{t\bar{t}} + \tilde{\mathcal{J}}_{t\bar{t}} + \tilde{\mathcal{J}}_\ominus)_\mu + 2\mathcal{J}_t^\mu \mathcal{J}_{\bar{t},\mu} \right] \right. \\ &\quad + \delta_{i''i'} \delta_{j'j''} \left[ N\mathcal{J}_t^\mu (\mathcal{J}_{t\bar{t}} + \tilde{\mathcal{J}}_{t\bar{t}} + \tilde{\mathcal{J}}_\oplus)_\mu + N\mathcal{J}_{\bar{t}}^\mu (\mathcal{J}_{t\bar{t}} - \tilde{\mathcal{J}}_{t\bar{t}} - \tilde{\mathcal{J}}_\ominus)_\mu \right. \\ &\quad \left. \left. - \frac{2}{N} (\mathcal{J}_t^\mu \mathcal{J}_{\bar{t},\mu} + \mathcal{J}_{\bar{t}}^\mu \mathcal{J}_{t\bar{t},\mu} + \mathcal{J}_{\bar{t}}^\mu \mathcal{J}_{t\bar{t},\mu}) \right] \right\} \\ &\quad + (Q_{\text{in}}^a)_{i''i';j'j''}^{c_2''c_2';c_1'c_1''} \left\{ \delta_{j'j''} (T^a)_{i''i'} \mathcal{J}_t^\mu \mathcal{J}_{\oplus,\mu} + \delta_{i''i'} (T^a)_{j'j''} \mathcal{J}_{\bar{t}}^\mu \mathcal{J}_{\ominus,\mu} \right\}, \end{aligned} \quad (7)$$

$$\begin{aligned} (\Delta_{\text{nf}}^{\text{real}})_{i''i';j'j''}^{c_2''c_2';c_1'c_1''} &= \frac{1}{2} \delta_{c_2''c_2'} \delta_{c_1'c_1''} \left\{ \delta_{i''j''} \delta_{j'i'} \left[ \mathcal{I}_t^{*\mu} (\mathcal{I}_{t\bar{t}} - \tilde{\mathcal{I}}_{t\bar{t}} - \tilde{\mathcal{I}}_0)_\mu + \mathcal{I}_{\bar{t}}^{*\mu} (\mathcal{I}_{t\bar{t}} + \tilde{\mathcal{I}}_{t\bar{t}} + \tilde{\mathcal{I}}_0)_\mu + 2\mathcal{I}_t^{*\mu} \mathcal{I}_{\bar{t},\mu} \right] \right. \\ &\quad + \delta_{i''i'} \delta_{j'j''} \left[ N\mathcal{I}_t^{*\mu} (\mathcal{I}_{t\bar{t}} + \tilde{\mathcal{I}}_{t\bar{t}} + \tilde{\mathcal{I}}_0)_\mu + N\mathcal{I}_{\bar{t}}^{*\mu} (\mathcal{I}_{t\bar{t}} - \tilde{\mathcal{I}}_{t\bar{t}} - \tilde{\mathcal{I}}_0)_\mu \right. \\ &\quad \left. \left. - \frac{2}{N} (\mathcal{I}_t^{*\mu} \mathcal{I}_{\bar{t},\mu} + \mathcal{I}_{\bar{t}}^{*\mu} \mathcal{I}_{t\bar{t},\mu} + \mathcal{I}_{\bar{t}}^{*\mu} \mathcal{I}_{t\bar{t},\mu}) \right] \right\} \\ &\quad + (Q_{\text{in}}^a)_{i''i';j'j''}^{c_2''c_2';c_1'c_1''} \left\{ \delta_{j'j''} (T^a)_{i''i'} \mathcal{I}_t^{*\mu} \mathcal{I}_{0,\mu} + \delta_{i''i'} (T^a)_{j'j''} \mathcal{I}_{\bar{t}}^{*\mu} \mathcal{I}_{0,\mu} \right\}. \end{aligned} \quad (8)$$

The terms proportional to  $\delta_{i''j''} \delta_{j'i'}$  project on the lowest-order singlet  $t\bar{t}$  states, whereas the terms proportional to  $\delta_{i''i'} \delta_{j'j''}$  completely factorize the lowest-order cross-section. The colour

structure

$$Q_{\text{in}}^a = \begin{cases} 0 & \text{for } e^+e^-, \gamma\gamma \\ \delta_{c'_1 c''_1}(T^a)_{c'_2 c''_2} + \delta_{c'_2 c''_2}(T^a)_{c'_1 c''_1} & \text{for } q\bar{q} \\ \delta_{c'_1 c''_1}(F^a)_{c'_2 c''_2} + \delta_{c'_2 c''_2}(F^a)_{c'_1 c''_1} & \text{for } gg \end{cases} \quad (9)$$

depends on the specific initial state and in general does not project on explicit lowest-order  $t\bar{t}$  colour states. Here  $F^a$  are the  $SU(N)$  generators in the adjoint representation, which are defined in terms of the  $SU(N)$  structure constant according to  $(F^a)_{bc} = -if^{abc}$ . Note that for  $e^+e^-$  and  $\gamma\gamma$  initial states the currents  $\tilde{\mathcal{J}}_\ominus^\mu$ ,  $\tilde{\mathcal{J}}_\oplus^\mu$  and  $\tilde{\mathcal{I}}_0^\mu$  completely drop out of Eqs. (5) and (6), as it should be for colourless particles in the initial state.

The semi-soft currents appearing in the virtual non-factorizable corrections are given by

$$\mathcal{J}_t^\mu = -g_s \left[ \frac{p_1^\mu}{kp_1 + io} - \frac{k_1^\mu}{kk_1 + io} \right] \frac{D_1}{D_1 + 2kp_1}, \quad \mathcal{J}_{\bar{t}}^\mu = -g_s \left[ \frac{p_2^\mu}{-kp_2 + io} - \frac{k_2^\mu}{-kk_2 + io} \right] \frac{D_2}{D_2 - 2kp_2} \quad (10)$$

for gluon emission from the decay stages of the process, and

$$\begin{aligned} \mathcal{J}_{t\bar{t}}^\mu &= g_s \left[ \frac{p_1^\mu}{kp_1 + io} + \frac{p_2^\mu}{-kp_2 + io} \right], & \tilde{\mathcal{J}}_{t\bar{t}}^\mu &= g_s \left[ \frac{p_1^\mu}{kp_1 + io} - \frac{p_2^\mu}{-kp_2 + io} \right], \\ \mathcal{J}_\oplus^\mu &= -g_s \left[ \frac{q_1^\mu}{kq_1 + io} - \frac{q_2^\mu}{kq_2 + io} \right], & \tilde{\mathcal{J}}_\oplus^\mu &= -g_s \left[ \frac{q_1^\mu}{kq_1 + io} + \frac{q_2^\mu}{kq_2 + io} \right], \\ \mathcal{J}_\ominus^\mu &= g_s \left[ \frac{q_1^\mu}{-kq_1 + io} - \frac{q_2^\mu}{-kq_2 + io} \right], & \tilde{\mathcal{J}}_\ominus^\mu &= g_s \left[ \frac{q_1^\mu}{-kq_1 + io} + \frac{q_2^\mu}{-kq_2 + io} \right] \end{aligned} \quad (11)$$

for gluon emission from the production stage of the process. Here  $g_s$  is the QCD gauge coupling and  $D_{1,2} = p_{1,2}^2 - m_t^2 + im_t\Gamma_t$  is a shorthand notation for the inverse top-quark propagators. Note the difference in the sign of the  $io$  parts appearing in the currents  $\mathcal{J}_\oplus, \tilde{\mathcal{J}}_\oplus$  and  $\mathcal{J}_\ominus, \tilde{\mathcal{J}}_\ominus$ . These infinitesimal imaginary parts are needed to ensure a proper incorporation of causality.

The corresponding semi-soft real-gluon currents read

$$\mathcal{I}_t^\mu = -g_s \left[ \frac{p_1^\mu}{kp_1} - \frac{k_1^\mu}{kk_1} \right] \frac{D_1}{D_1 + 2kp_1}, \quad \mathcal{I}_{\bar{t}}^\mu = g_s \left[ \frac{p_2^\mu}{kp_2} - \frac{k_2^\mu}{kk_2} \right] \frac{D_2}{D_2 + 2kp_2} \quad (12)$$

and

$$\begin{aligned} \mathcal{I}_{t\bar{t}}^\mu &= g_s \left[ \frac{p_1^\mu}{kp_1} - \frac{p_2^\mu}{kp_2} \right], & \tilde{\mathcal{I}}_{t\bar{t}}^\mu &= g_s \left[ \frac{p_1^\mu}{kp_1} + \frac{p_2^\mu}{kp_2} \right], \\ \mathcal{I}_0^\mu &= -g_s \left[ \frac{q_1^\mu}{kq_1} - \frac{q_2^\mu}{kq_2} \right], & \tilde{\mathcal{I}}_0^\mu &= -g_s \left[ \frac{q_1^\mu}{kq_1} + \frac{q_2^\mu}{kq_2} \right]. \end{aligned} \quad (13)$$

By simple power counting one can explicitly see from the above specified currents that the contributions of hard gluons are suppressed and that effectively only semi-soft gluons with  $E_g = k_0 = \mathcal{O}(\Gamma_t)$  contribute. In view of the pole structure of the virtual corrections, governed by

the infinitesimal imaginary parts  $i0$ , many of the non-factorizable corrections will vanish when virtual and real-gluon corrections are added up. For instance, all initial-final state interferences will vanish, leaving behind a very limited subset of ‘final-state’ interferences [10, 16]. The following holds for the remaining interferences:

$$\mathcal{I}_t^* \mu \tilde{\mathcal{I}}_{t\bar{t},\mu} \rightarrow -\mathcal{I}_t^* \mu \mathcal{I}_{t\bar{t},\mu}, \quad \mathcal{I}_{\bar{t}}^* \mu \tilde{\mathcal{I}}_{t\bar{t},\mu} \rightarrow \mathcal{I}_{\bar{t}}^* \mu \mathcal{I}_{t\bar{t},\mu},$$

with similar effective replacements for  $\tilde{\mathcal{J}}_{t\bar{t}}$ . As a result of these properties of the non-factorizable corrections, a factorization per colour structure emerges:

$$d\sigma_{\text{nf}} = \delta_{\text{nf}} \left[ \frac{N^2 - 1}{2N} d\sigma_{\text{Born},1} - \frac{1}{2N} d\sigma_{\text{Born},8} \right], \quad (14)$$

$$\begin{aligned} \delta_{\text{nf}} = 2\text{Re} \left\{ i \int \frac{d^4 k}{(2\pi)^4 [k^2 + i0]} \left[ \mathcal{J}_t^\mu \mathcal{J}_{\bar{t},\mu} + \mathcal{J}_t^\mu \mathcal{J}_{t\bar{t},\mu} + \mathcal{J}_{\bar{t}}^\mu \mathcal{J}_{t\bar{t},\mu} \right] \right. \\ \left. - \int \frac{d\vec{k}}{(2\pi)^3 2k_0} \left[ \mathcal{I}_t^* \mu \mathcal{I}_{\bar{t},\mu} + \mathcal{I}_t^* \mu \mathcal{I}_{t\bar{t},\mu} + \mathcal{I}_{\bar{t}}^* \mu \mathcal{I}_{t\bar{t},\mu} \right] \right\}. \end{aligned} \quad (15)$$

Here  $d\sigma_{\text{Born},1}$  and  $d\sigma_{\text{Born},8}$  are the lowest-order multi-differential cross-sections for producing the intermediate  $t\bar{t}$  pair in a singlet and octet state, respectively. For completeness we note that

$$d\sigma_{\text{Born}}^{e^+e^-, \gamma\gamma} = d\sigma_{\text{Born},1}^{e^+e^-, \gamma\gamma}, \quad d\sigma_{\text{Born}}^{q\bar{q}} = d\sigma_{\text{Born},8}^{q\bar{q}}, \quad d\sigma_{\text{Born}}^{gg} = d\sigma_{\text{Born},1}^{gg} + d\sigma_{\text{Born},8}^{gg}. \quad (16)$$

The non-factorizable factor  $\delta_{\text{nf}}$  can be obtained from Ref. [13]. The results of Section 4 of that paper should be used, since those allow for massive decay products from the unstable particles, which is the case for the top-quark decay.

We conclude by considering the case that also the  $W$  bosons are unstable. This adds two decay subprocesses,  $W_{\text{dec}}^+$  and  $W_{\text{dec}}^-$ , to the three we have considered so far. If the  $W$  bosons decay leptonically, nothing changes as the gluon cannot couple to the  $W$  decay subprocesses in that case. For a hadronically decaying  $W$  boson additional interferences have to be taken into account. However, such interferences trivially vanish as a result of the singlet nature of the  $W$ -boson decays [i.e.  $\text{Tr}(T^a)=0$ ].

## 4 Numerical results

With the help of Eqs. (14)–(16) we can now in principle evaluate all kinds of multi-differential distributions, with and without non-factorizable corrections. Although the factorized structure of the non-factorizable corrections is very transparent in Eq. (14), integration of the multi-differential cross-sections will affect this structure. For instance, in Eq. (14) the correction to the singlet cross-section differs by a factor  $-8$  with respect to the octet one. However, for the calculation of the relative non-factorizable corrections to a one-dimensional distribution, one has to evaluate the ratio of the integrated Eqs. (14) and (16). Since  $\delta_{\text{nf}}$  depends on the integration variables, the thus-obtained singlet and octet correction factors will not necessarily differ by the factor  $-8$ .

At this point we stress that any observable that is inclusive in both top-quark invariant masses, such as the total cross-section, will not receive any non-factorizable corrections. This is a typical feature of these interconnection effects [16]. As an example of a distribution that is subject to non-vanishing non-factorizable corrections we focus on the invariant-mass distribution of the top quark, which can be used for the mass determination. To this end we determine the non-factorizable correction  $\delta_{\text{nf}}(M)$  for the distribution

$$\frac{d\sigma}{dM} = \frac{d\sigma_{\text{Born}}}{dM} [1 + \delta_{\text{nf}}(M)], \quad (17)$$

where  $M$  is the invariant mass of the  $b$ -quark and the  $W^+$  boson. The maximum of the Breit–Wigner distribution can be used to determine the top-quark mass. The linearized shift of this maximum as induced by the non-factorizable corrections is given by

$$\Delta M = \frac{1}{8} \Gamma_t^2 \left. \frac{d\delta_{\text{nf}}(M)}{dM} \right|_{M=m_t}. \quad (18)$$

The correction  $\delta_{\text{nf}}(M)$  is calculated for the four different mechanisms of  $t\bar{t}$  production, i.e. initiated by  $e^+e^-$ ,  $\gamma\gamma$ ,  $q\bar{q}$  and  $gg$ . For the centre-of-mass energies of these (partonic) reactions we take  $\sqrt{s} = 355 \text{ GeV}$  and  $500 \text{ GeV}$ . These values exemplify the non-factorizable corrections in the vicinity of the threshold and far above it. As mentioned before, the adopted approximation in our calculation (LPA) forces us to stay sufficiently far above the  $t\bar{t}$  threshold (read: a few times  $\Gamma_t$ ). The numerical values for the input parameters are

$$m_t = 173.8 \text{ GeV}, \quad M_W = 80.26 \text{ GeV}, \quad M_Z = 91.187 \text{ GeV}, \quad (19)$$

and

$$\Gamma_t = 1.3901 \text{ GeV}, \quad (20)$$

the latter being the  $\mathcal{O}(\alpha_s)$  corrected top-quark width. The correction  $\delta_{\text{nf}}$  is proportional to  $\alpha_s$ , for which we have to choose the relevant scale. For  $\sqrt{s} = 355 \text{ GeV}$  the main contribution originates from the non-factorizable Coulomb effect present in  $\delta_{\text{nf}}$ . Its typical momentum is determined by the top-quark width  $\Gamma_t$  and velocity  $\beta$ :  $\Gamma_t/\beta \sim 6.8 \text{ GeV}$ . At  $500 \text{ GeV}$  softer gluons contribute and therefore the typical gluon momentum is  $\Gamma_t \sim 1.4 \text{ GeV}$ . Therefore we choose

$$\alpha_s(1.4 \text{ GeV}) \approx 0.3536 \quad \text{for} \quad \sqrt{s} = 500 \text{ GeV}, \quad (21)$$

$$\alpha_s(6.8 \text{ GeV}) \approx 0.1955 \quad \text{for} \quad \sqrt{s} = 355 \text{ GeV}, \quad (22)$$

corresponding to  $\alpha_s(M_Z) = 0.1180$  at the  $Z$  peak. It should be noted that choosing another scale in  $\alpha_s$  will only affect the normalization of the correction.

In Fig. 2 the non-factorizable correction  $\delta_{\text{nf}}$  is plotted as a function of the invariant mass  $M$  at the centre-of-mass energy of  $355 \text{ GeV}$ . The  $\delta_{\text{nf}}$  values for the pure singlet  $e^+e^-$  initial state and the pure octet  $q\bar{q}$  initial state differ approximately by the afore-mentioned factor of  $-8$ . For the  $gg$  initial state the Born octet part is larger than the singlet one, resulting in a non-factorizable correction that lies between the  $e^+e^-$  and the  $q\bar{q}$  case. The correction for the



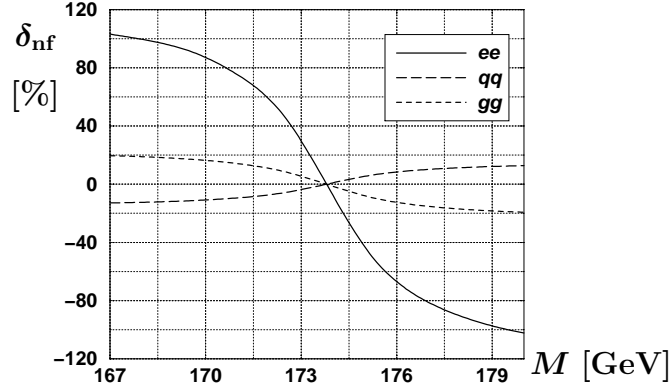


Figure 2: The relative non-factorizable correction  $\delta_{nf}(M)$  to the single invariant-mass distribution  $d\sigma/dM$ . Centre-of-mass energy:  $\sqrt{s} = 355$  GeV.

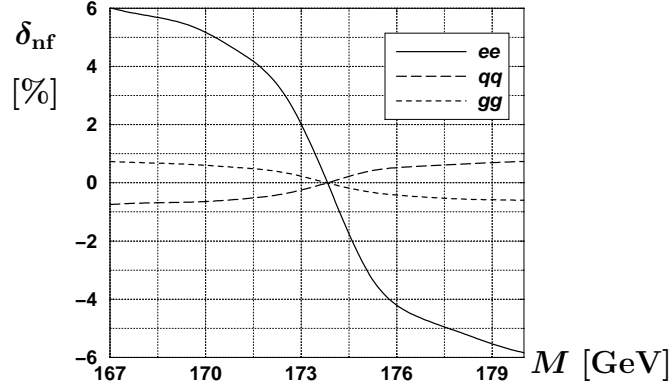


Figure 3: The relative non-factorizable correction  $\delta_{nf}(M)$  to the single invariant-mass distribution  $d\sigma/dM$ . Centre-of-mass energy:  $\sqrt{s} = 500$  GeV.

$\gamma\gamma$  initial state is virtually indistinguishable from the  $e^+e^-$  one and is therefore not displayed. Evidently the distortion effects from the singlet corrections are very large, which is due to a large non-factorizable Coulomb correction inside  $\delta_{nf}$ . The maximum of the Breit–Wigner distribution is hardly affected by this large correction. One finds for the various initial states  $e^+e^-(\gamma\gamma)$ ,  $gg$  and  $q\bar{q}$   $\Delta M \approx -85$ ,  $-15$  and  $+10$  MeV respectively. The situation at 500 GeV is depicted in Fig. 3. The overall correction is small, which is typical for non-factorizable corrections further away from threshold. The shift in the maximum of the Breit–Wigner distribution is of the order of 5 MeV for the  $e^+e^-$  and  $\gamma\gamma$  initial states, and even smaller for the  $q\bar{q}$  and  $gg$  initial states.

In order to obtain hadronic distributions from the partonic ones, the results for the  $q\bar{q}$  and  $gg$  initial states should of course be properly folded with the parton densities of the colliding hadrons ( $p\bar{p}$  at the Tevatron,  $pp$  at the LHC). The bulk of the partonic contributions originates from the energy region not far above the  $t\bar{t}$  threshold ( $s \lesssim 8m_t^2$ , i.e.  $\sqrt{s} \lesssim 500$  GeV), which is exemplified by the partonic energies 355 and 500 GeV used in our analysis.

## 5 Conclusions

In this paper we have summarized the gauge-invariant description for calculating the  $\mathcal{O}(\alpha_s)$  non-factorizable QCD corrections to pair production of top quarks. The formalism is presented in a general way, making it applicable to all relevant initial states. The resulting final formula for the non-factorizable corrections involves the same quantity  $\delta_{\text{nf}}$  for all reactions. This quantity can be numerically calculated using expressions available in the literature.

Although the formalism can be used for numerical studies of many distributions, the focus of our numerical evaluation has been on the invariant-mass distribution of the top quark, which can be used for extracting the top-quark mass. In spite of the possible sizeable deformations of this line-shape distribution, its maximum is shifted by less than 100 MeV. Therefore, if the top-quark mass is extracted experimentally from the peak position of the line-shape, the non-factorizable corrections can be safely neglected. If the precise shape of the Breit–Wigner distribution is used in the experimental analysis, the non-factorizable corrections should be taken into account properly. In particular if the singlet colour state dominates. In addition higher-order non-factorizable corrections might be needed.

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